CS70

Public Key Cryptography (RSA)

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Overview

1. Introduction to RSA

2. RSA Scheme

Intro to RSA

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Basic setting:

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- Bob, after receiving $E(x)$, applies his **decryption function** D to it and recover the original message: i.e., $D(E(x)) = x$.
- Since the link is insecure, Eve may know what $E(x)$ is.

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- The idea is that each person has a **public key** known to the whole world and a **private key** known only to him- or herself.
- Alice encodes x using Bob's public key. Bob then decrypts it using his private key, thus retrieving x.

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- Treat messages to Bob as numbers modulo N, excluding trivial values 0 and 1.
- Let e be any number that is relatively prime to $(p-1)(q-1)$ (Typically e is a small value).
- Then Bob's public key is the pair of numbers (N, e) and his private key is $d = e^{-1} \pmod{(p-1)(q-1)}$.

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- **Encryption**: Alice computes the value $E(x) = x^e \bmod N$ and sends this to Bob.
- **Decryption:** Upon receiving the value $y = E(x)$, Bob computes $D(y) = y^d \bmod N$; this will be equal to the original message x.

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Using the encryption and decryption functions E and D, we have $D(E(x)) = x \pmod N$ for every possible message $x \in \{0, 1, ..., N-1\}$.

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Proof:

This can be proved using Chinese Remainder Theorem or Fermat's Little Theorem. For more details, please refer to notes. □

Problem Time!