CS70

Discrete Probability

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Overview

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- **sample space** Ω : set of all all possible outcomes of an experiment.
- **probability law:** assigns to a set A of possible outcomes (event) a nonnegative value $P(A)$ (probability of A) that encodes the knowledge about the likelihood of the elements of A.

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A subset of the sample space.

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- **Additivity:** any countable sequence of **disjoint sets** (*mutually exclusive* events) $E_1, E_2, \ldots$ satisfies

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- If $A \subseteq B$, then $P(A) \leq P(B)$.

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For uniform spaces, computing probabilities is simply counting sample points.

Problem Time!